American University of Beirut
Faculty of Engineering & Architecture
Department of Civil & Environmental Engineering

CIVE 441 Engineering Hydrology **Fall 2009**
Manal Moussallem **January 29, 2010**

FINAL

I pledge my honor that I will not give or receive any assistance on this exam

NAME: **ID:**

This is a closed book, closed notes 3-hr exam (8 problems equally weighted). The use of non-programmable calculators is allowed. Write clearly☺

Problems 1 to 6 relate to the same case; however, they are independent of each other.

Following your graduation from AUB in summer 2011, you are hired by a reputable engineering consulting firm, which has been awarded a contract for the sustainable management of the Nahr El Kalb watershed, Mount Lebanon. A key component of the project is the hydrological modelling of the watershed (~250 km^2 ; i.e. ~ 95 mi^2)[1]. The project director assigns this activity to a team of three, including you.

PROBLEM 1: Getting familiar with the watershed balance...
Your first task is to get familiar with the watershed through site visits, literature review, meetings with stakeholders, etc. While undertaking this task, you come across a document that states the following:

The Nahr El Kalb watershed received a total of 80 cm of precipitation in 1993. The average rate of flow measured in a gage at the outlet of the watershed was 3 m^3/s.

You decide to use this data to develop a preliminary watershed balance:

1. Estimate the water losses during the year (in cm) due to the combined effects of evaporation, transpiration, and groundwater infiltration.
2. What is the total runoff (in cm) which reached the river during that year?

[1] 1 mile is 1.609 km

At the biweekly progress meeting of the team, you present the preliminary watershed balance (problem 1) and a set of panoramic photos that will help your colleagues see the overall picture. You get the following two questions from the floor, which you have to prepare for the next progress meeting:

PROBLEM 2: Analyzing rainfall data

One of your colleagues points out to you that it is very important to statistically analyze rainfall data, to support the methodology and choice of data used. He specifically asks you to compute the following four values, assuming that the statistics of the log(10) transformed annual total rainfall collected over a 21-yr period for Nahr El Kalb are: mean = 1.6637, standard deviation = 0.0993, and corrected skewness = -1.6:

2-1) the return period of 80 cm (i.e. the value provided in problem 1), using the lognormal distribution

2-2) the 25-yr annual total rainfall (in cm) using the Log-Pearson 3 distribution

2-3) the probability that the annual rainfall will fall between 70 cm and 80 cm exactly once in the five consecutive years, using the lognormal distribution.

2-4) the risk of the 50-yr annual rainfall occurring in the next 10 years, using the Log Pearson 3 distribution

PROBLEM 3: What about dewatering?

A student/ trainee in the company, who lives in Nahr El Kalb area, reports some construction works in the area (as per below):

*Two wells are installed on the corner of a square 100 m * 100 m in order to dewater the foundation pit. The 2 wells are located at the coordinates (x; y) = (0; 0); (x;y) = (100;100) and the river is along the y-axis at x=200m. The objective is to drop the water level by 50 cm at the centre of the square, in 2 days.*

Compute the necessary pumping rate per well (in m^3/min), assuming:

- Equal pumping rates at the two wells
- The transmissivity is 3 m^2/min
- The storativity is 0.15

Extra Credit: Does this affect the water balance?

At the second bi-weekly progress meeting, you present your answers for the two questions raised. You then share with the team the data you collected from the Ministry of Public Works – Directorate General of Civil Aviation, the Ministry of Energy and Water – Litani River Authority and other sources, which will help you initiate your modelling exercise:

- *Cumulative Rainfall on January 2, 2008; 12 noon – 3PM*

Time (hr)	*0*	*1*	*2*	*3*
$\sum P$ (in)	*0*	*0.5*	*4*	*4*

- *Observed River Outflow on January 2, 2008; starting 1 PM*

T (hr)	*0*	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*
Q (cfs)	*0*	*1,635*	*5,535*	*8,865*	*12,420*	*16,585*	*17,595*	*15,040*	*12,880*	*10,675*	*8,475*

- *Watershed characteristics:*
  - *Length of the river: 8 miles (footnote 2)* [2]
  - *Slope: 2%*
  - *Curve number: 80 (based on a mostly agricultural land use)*

PROBLEM 4: Calculating the excess rainfall hyetograph

You start by determining the excess rainfall hyetograph for the January 2008 storm above, using the SCS method with the curve number above and normal antecedent moisture conditions.

Compute and draw the excess rainfall hyetograph, by outlining your methodology first.

[2] 1 mile is 5,280 ft

PROBLEM 5: Generating the runoff

You then generate the runoff corresponding to the excess rainfall hyetograph (computed in problem 4), using the SCS method (and the watershed characteristics outlined before).

Compute and draw the generated runoff by outlining your methodology first.

PROBLEM 6: Routing the runoff

You then route the runoff (generated in problem 5), using the Muskingum method, with Δt=60 min and zero initial conditions. The Muskingum parameters of the reach are x = 0.2 and k = 30 min

How does the flow at the outlet compare with the observed flow given on page 7?

<u>PROBLEM 7:</u> How will the hydrologist help?

7-1 The Lebanese Company for the Development and Reconstruction of Beirut Central District (SOLIDERE), in coordination with the Council for Development and Reconstruction, has decided to find an alternative location for the wastes that have been deposited at the reclaimed sea area in Beirut for more than 20 years. The Sebline abandoned quarry was identified as a suitable new location for these wastes, which will be used to rehabilitate this quarry. The contractor, who has been awarded this job, was asked by the Ministry of Environment to undertake an environmental impact assessment (EIA) for this project (i.e. identify the potential impacts of this project on the environment and design the necessary measures to reduce the negative impacts). Why is a hydrologist needed in the preparation of the EIA study/ what task will be assigned to him?

7-2 The United Nations Development Programme (UNDP) has been awarded a 1.8 million Euro grant from the Italian Cooperation to update the water balance of Lebanon, in close coordination with the Ministry of Energy and Water. UNDP is looking for an expert to manage this project. You apply for the position and take a written exam where you are asked to develop a draft proposal – i.e. you have to list the main activities of such a project.

Extra-credit: can you estimate the time-frame and budget of each of the activities?

PROBLEM 8: Meet George F. Pinder...

Read the excerpts copied from the book titled *A Civil Action* by Jonathan Harr, and answer the question below. *A Civil Action* follows the story of a law suit brought by leukemia (cancer) victims against two Woburn corporations for contaminating municipal wells with TCE (trichloroethylene[3]).

8-1) If George Pinder corrected his estimate of arrival time from 12 months to 9 months, what was his estimate of porosity?

8-2) What do you think will be George Pinder's realization about the cause of the decline in depth of the Aberjona river?

8-3) What simple field data could be used to check Pinder's hypothesis?

Extra Credit: Where will Pinder have his realization?
Recommendation: Watch the movie *A Civil Action* starring John Travolta

Enjoy the break and the CES Ski Trip ☺

[3] The chemical compound trichloroethylene is a chlorinated hydrocarbon commonly used as an industrial solvent

They would have to send Helen O'Connell to a second urologist, continued Phillips, and perhaps even another neurologist, for a new round of complete workups. "If that doesn't convince them," Phillips told Schlichtmann, "we'll have to put the case on the trial calendar for next fall and hope you're finished with Woburn by then. We can't sell out. We've got to settle for fair value or we won't settle."

Gordon groaned again.

After eight weeks of trial, the days in the courtroom had begun to blur into one another, like the countryside seen from a train window. A shaft of spring sunlight would find its way into the court and strike the brass lamp on the judge's bench, a flash of brilliant yellow in the cavernous, gray, dismal room. The fluorescent lights overhead made everyone appear pale and sickly. Thick volumes of legal papers had grown on the counsel tables and more thick volumes lay underfoot in cardboard boxes on the floor. The lawyers' overcoats, damp from a morning rain and smelling of wool, hung over the gallery railing. The radiators hissed gently. Distant, muted sounds of city traffic, a siren, an unmuffled truck, would float up into the courtroom from the streets fifteen stories below. The atmosphere felt heavy and dense. One of the alternate jurors regularly fell asleep. On particularly dull days, such as the one when Schlichtmann read Grace's answers to interrogatories into evidence, Judge Skinner himself seemed to doze at the bench, the flesh of his cheeks slack and his mouth slightly parted, his head rolling back onto his chair.

Facher always used to warn his students at Harvard that if they fell asleep at the counsel table, upon awakening they should come to their feet objecting. "In the time it takes to reach your full height, think of a reason for your objection."

Facher was being facetious, of course. He never really expected any of his students to fall asleep at the counsel table. But one day, eight weeks into the trial, Facher himself fell into a doze during Sandra Lynch's slow and methodical examination of a Grace executive. He awoke when the microphone on the witness stand emitted a harsh squeal.

"Perhaps we can fix the microphone," suggested Lynch.

"Mr. Nesson seems to have the touch," said the judge.

Facher came to his feet at the mention of Nesson's name. "I object," Facher said in a thick, cottony voice. Then he looked around, blinking his eyes, and sat back down.

5

On a morning in early May, Kathy Boyer opened the windows in the conference room for the first time since last fall, and there was fresh air in the offices of Schlichtmann, Conway & Crowley. Across from the courthouse, in Post Office Square, the tulips had burst into flower and the swollen buds on the linden trees had split open into a lacy green filigree of new leaves. Schlichtmann noticed that it was spring, but that didn't cheer him. This part of the trial should have ended long before tulips.

On the other hand, he was making progress. He had one final witness to call in this phase of the trial. This was his expert in hydrology and groundwarer movement, one George E. Pinder, Ph.D. Pinder would testify against both Grace and Beatrice. He would, if all went well, make manifest the claim that TCE and other solvents on the properties of both defendants had indeed migrated to the city wells, and that the chemicals had gotten there before the leukemias and other illnesses began occurring.

Schlichtmann felt fortunate to have Pinder. Every geologist who knew anything about groundwater had heard of Pinder. He was preeminent in his field, chairman of the civil engineering department at Princeton University and the person who, fifteen years ago, had developed the first computer model of groundwater flow. Schlichtmann put him up at the Ritz-Carlton, in a suite of rooms.

Pinder was in his late forties, a dapper, diminutive man, nearly a foot shorter than Schlichtmann. His thin brown hair, as soft and silky as a baby's, had receded high on the dome of his head. He wore gold-rimmed spectacles that made his round face look owlish and cerebral. He had a precise, methodical manner, but he was not in the least aloof or self-important. On the morning of his first day in court, he arrived at Schlichtmann's office early and cordially greeted all the secretaries. Schlichtmann saw that Pinder was wearing a blue blazer, a woolly brown tie, brown pants and argyle socks. Schlichtmann always asked his experts to dress conservatively, in dark suits. To his eyes, Pinder was

a sartorial nightmare, but he took it calmly. "George is the world's leading expert on groundwater," he told Conway privately before they left for court. "He can get away with dressing like that."

Pinder performed ably on his first day, under direct examination by Schlichtmann. He came to the courtroom equipped with charts and diagrams to educate the jury in the science of hydrogeology. He set up a fish tank filled with kitchen sponges and placed drops of ink on the sponges. "Just think of each drop as being some event of contamination on the ground, entering our aquifer," Pinder told the jury. He talked about saturated and unsaturated zones, capillary fringes, cones of depression. He held forth for the entire morning, confident, jaunty, and full of good humor, as if he were lecturing to a freshman class at Princeton.

Both Facher and Cheeseman knew Pinder's reputation. Two years ago, Cheeseman himself had tried to recruit Pinder as an expert witness for Grace, but he found that Schlichtmann had gotten to Princeton, and to Pinder, first. Facher wasn't worried, though. "They tell me Pinder's the leading expert on this subject," remarked Facher out in the corridor, during the morning break. "They say he's a home-run hitter in any ballpark. But he's in *my* ballpark now."

Indeed, things did not go quite so smoothly after the recess, although that was no fault of Pinder's. The closer Schlichtmann brought Pinder to implicating Beatrice and Grace, the more Facher and Keating began to object. Nevertheless, by the end of the day, Schlichtmann had gotten into evidence the first part of Pinder's opinion—that both Beatrice and Grace were responsible for the contamination of Wells G and H. The second part, the arrival times of TCE and the other solvents, would have to wait until tomorrow.

In the office that afternoon, Schlichtmann clapped his hands and danced on his toes in glee. "You got the opinion in, George!" he shouted. "They tried to stop you, but it didn't happen. It was a great day, today, George! It could not have gone better!"

Pinder smiled indulgently. "This jury looks like a pretty attentive group," he said.

Pinder made his first mistake the next morning. It was a small mistake, but Schlichtmann caught it in an instant. It happened when he asked Pinder how long it would take, in his opinion, for TCE and the three other solvents to migrate from the Beatrice and Grace sites to the wells. Schlichtmann had gone over these calculations with Pinder the night before, but that morning, Pinder gave different times for the solvents, different by a matter of days for one, a few weeks for another, and a year for the third.

Schlichtmann wondered for a moment if he himself was wrong, if he had remembered the times incorrectly. He felt confused, but he had no chance to find out at that moment why Pinder had changed the times. And when the recess finally came, Schlichtmann decided not to broach the subject. He did not want to shake Pinder's confidence in the middle of the day.

That evening, in Pinder's room at the Ritz-Carlton, Schlichtmann found out what had gone wrong. Pinder, feeling overly confident, redid the calculations in his head and forgot to factor in the porosity of the soil. The mistake made no difference to the substance of Pinder's opinion. The solvents had still reached Wells G and H long before the leukemias began occurring. But Schlichtmann knew that Facher and Keating would not miss this mistake, and that they would use it on cross-examination to attack Pinder's credibility.

Schlichtmann decided to wait until the closing minutes of the next day, a Friday, to have Pinder correct the mistake. He'd make it appear as almost an afterthought, of small consequence but requiring mention nonetheless. That would give the jurors the entire weekend to digest the substance of Pinder's opinion before cross-examination could begin on Monday morning.

The next day all went as planned for a while. When only twenty minutes remained before court adjourned for the weekend, Schlichtmann brought up the travel times, and Pinder explained that yesterday, on the witness stand, he had done the calculations in his head. "I was just contemplating my testimony," continued Pinder in a pleasant, untroubled voice, "and it suddenly occurred to me that I'd made a mistake. I'd be very pleased to try and correct that mistake."

"What was the mistake you made?" asked Schlichtmann.

"I left off the constant for the porosity of the soil when I was doing the multiplication in my head."

"Does that affect the travel times in some way?"

"It affects the travel times," said Pinder, nodding. "Not catastrophically but, I think, significantly."

Schlichtmann asked Pinder to tell the jury the new figures. When Pinder finished doing so, Schlichtmann glanced at the clock on the back wall of the courtroom, above the gallery. He still had twelve minutes to occupy until court recessed for the weekend. He planned to end the day by bringing out the fact that the research done in east Woburn—twelve thousand pages of data, 157 monitoring wells, and dozens of volumes from technical consultants—made the Aberjona aquifer one of the most thoroughly studied aquifers in history. He asked Pinder to compare the east Woburn research with other projects that Pinder had worked on, but both Facher and Keating objected. "Sustained," said the judge. Schlichtmann rephrased his question, but again the judge sustained the objections.

"May I have a moment, Your Honor?" Schlichtmann asked.

The judge nodded. Schlichtmann took a deep breath. He studied his notes. He needed another question but he couldn't think of one. He went to the counsel table and bent down to consult with Nesson, who hurriedly scribbled out a question. Schlichtmann turned and asked the question, but Facher objected, the judge sustained the objection, and Schlichtmann gave up. There were only seven minutes remaining. He said, "No more questions, Your Honor."

The judge peered at the clock. "This is probably a good place to stop, since we will begin cross-examination on Monday morning."

The little skirmishes of lawyers are sometimes consequential. Schlichtmann's strategy was obvious enough, and Facher had no intention of letting him get away with it. Facher stood and said to the judge, "Would you give me the seven minutes?"

"You want to start your cross-examination now?" said the judge, looking surprised and not particularly happy.

"Yes," said Facher, his eyes on Pinder as he walked across the well of the courtroom.

Pinder flew home to Princeton that Friday evening. After his seven minutes with Facher, he didn't relish the prospect of returning to Boston on Monday. Facher had treated him in a most contemptible manner, addressing him in an insulting and scornful tone that Pinder, for one, had never before experienced in his adult life. "Are you telling this jury that you came in here yesterday, as a Ph.D. and the chairman

of a department, and made a *little* mistake in an opinion you've been preparing for the last year and a half?" Facher had said. "You're telling us, as a professor of geology, that you *forgot* to take into account *porosity?* Didn't you lecture in front of this jury for an hour about making these calculations? Today is true and yesterday was not? That is what you want this jury to believe?"

Pinder felt he'd kept his wits and replied calmly, but the brief ordeal had shaken him. He had testified before, in the Love Canal and Velsicol cases, but on those occasions he'd been on the witness stand for only a short time and his opinions had gone virtually unchallenged. Pinder's wife, Phyllis, took an interest in her husband's work. She knew about Facher from reading the trial transcripts of Drobinski's testimony, which Schlichtmann had sent down for Pinder to peruse. "Watch out for Facher," she warned her husband before he returned to Boston. "You should read your deposition so you won't contradict yourself."

Pinder didn't take his wife's advice. His deposition had gone on for five days and amounted to almost a thousand pages. He didn't bother to read it, but Facher did. Facher read every page.

On Monday morning Facher asked Pinder if he recalled saying at his deposition that the contaminants from Beatrice would have reached the wells within eighteen months. Pinder replied that he didn't remember exactly what he'd said. "But I think that is reasonable, and what I was likely to have said."

"When you testified here in court last week, you said the contaminants had reached the well field within a year. Do you remember that?"

"I don't remember the details," said Pinder. "But if you say that's what I said, I'll accept that."

"You don't consider that a change from eighteen months?"

Pinder replied slowly, choosing his words with care. "It depends on what context I was thinking of the word 'contaminants' when you were using it. That is why it's a little difficult for me to try to be more precise."

Facher suggested that Pinder had formed his opinion before even seeing any data from the pump test. Pinder denied this.

"But you had a hypothesis as to the source of the contamination, right?" asked Facher.

Pinder, thinking that he might have said something like that at his deposition, replied, "I think that is not an unreasonable statement. I think I probably would be prepared to say that I may have said that."

"You *may* be prepared to say that you *may* have said that?" repeated Facher in an incredulous voice.

"Well, I'm a cautious man," said Pinder.

"Very cautious," said Facher. "You use words carefully, right?"

"I try to be as precise and accurate as I can," said Pinder.

Pinder's attempt to be precise and accurate led to dense thickets of confusion and imprecision. Pinder was wary of Facher. He looked for a trap in every question Facher asked. To avoid being trapped, he refused to answer even the simplest questions in a simple way. When Facher asked him about Drobinski's work on the fifteen acres, Pinder said, "I'm not really familiar with what he did in detail. I think in spirit he went back and found some additional things."

"In *spirit* he went back?" said Facher in a mocking voice.

"In the spirit of your question, he went back," replied Pinder. "I have no particular, precise knowledge of the whole matter."

"You didn't even know who Mr. Drobinski was back in December of 1985, did you?"

"Oh, yes, I knew who he was," said Pinder with certitude. "We had talked together many times."

Facher picked up Pinder's deposition. He opened it and flipped through the pages until he found what he was looking for. "At your deposition on December tenth, I asked if you had worked directly with any Weston geologist, and you said yes. I asked, 'Can you identify them by name?' " Facher, standing near the witness stand now, placed the deposition in front of Pinder. "What was your answer?" Facher asked, pointing to the line he wanted Pinder to read.

Pinder leaned over his deposition and adjusted his spectacles. Facher gazed at the ceiling. It took Pinder a long time to answer. He was reading, it seemed, the entire page. " 'No,' " Pinder read aloud at last.

"You wouldn't have known Mr. Drobinski unless he stood in front of you with one of those little 'Hello, I'm Mr. Drobinski' tags on him?"

"At that time I didn't know who he was," said Pinder. "I'd spoken to him. There were several people, and I couldn't distinguish one from the other. That is the spirit of my answer."

"That's the spirit and the fact of your answer?"

Pinder soon abandoned "the spirit" and adopted new phrases. Everything became "in the context of what you're talking about," or "in the sense of what you're asking me." Facher didn't let these slip by. "I didn't put any sense in the question," he told Pinder. "I just asked a simple question."

The judge called the lawyers up to his bench. He said to Schlichtmann, "I'm beginning to get the impression that this fellow has either got a very loose grasp of the language, or he will say anything that comes into his head."

"I don't think that's a fair characterization of his testimony," said Schlichtmann, who knew perfectly well that it was going very badly.

After court that day, Conway saw Schlichtmann alone in his office, sprawled on the couch. Schlichtmann's arm covered his face as if he were shielding his eyes from a bright light.

"Boy, you look like shit," Conway said, standing in the doorway.

Schlichtmann lifted his arm from his eyes and glanced up at Conway. "This is going to be the worst fucking week of my life."

Conway nodded. "What the judge said about Pinder was very disturbing."

"That arthritic old bastard," murmured Schlichtmann.

"There's nothing worse than watching your witness being raped," said Conway. "It's awful to sit there and not be able to do anything."

"Are we going to survive the week?" asked Schlichtmann. "Four more days of this?" He gave a weak, dispirited laugh.

"We'll survive, Jan," Conway said, playing his part once again. He hitched up his pants. "George is the guru, the world's main expert. He knows more about that aquifer than anyone else in the world." Conway paused, and then he added, "Besides, I don't think anything could be worse than today."

At this, Schlichtmann sat up. He looked soberly at Conway. "Do you think it was really that bad?" Schlichtmann laughed again, the same weak laugh. "George actually told me he felt good today. Can you believe it? Ah, it's not George's fault. He's a brilliant guy, but he's not the sort of person who can move others. It's just not the way he is."

Gordon and Phillips walked into Schlichtmann's office. Gordon settled his heavy frame in the chair behind Schlichtmann's desk, put his feet up and lit a cigarette. Phillips sat in the armchair next to the couch and tried his hand at cheering up Schlichtmann. "Facher's little clinic today was great for lawyers, but it doesn't mean shit with the jury."

Phillips hummed nervously. "Just remember, Jan, the biggest victories are won by the slimmest margins."

There was a moment of silence. Everyone seemed to ponder this bit of wisdom. Finally Gordon said, "What exactly does that mean, Mark?"

Schlichtmann departed for the Ritz-Carlton and an evening of work with Pinder. He consoled himself by reasoning that although Facher might have tarnished Pinder's credibility, Facher had not succeeded in damaging the substance of Pinder's opinion. Schlichtmann felt he could make Pinder shine again on redirect.

Meanwhile, Schlichtmann had other concerns. He knew that Facher would try to use the Aberjona River, which flowed between the Beatrice property and the city wells, as a defense. According to Facher's theory, the pumping action of the wells would draw water directly out of the river, satisfying the wells' demand while at the same time blocking the flow of contaminated groundwater from Beatrice.

This theory had some merit. A year ago last spring, Pinder himself had warned Schlichtmann that the river might be "a very profound barrier," although Pinder personally doubted this would prove true. His computer model of the east Woburn aquifer predicted that the city wells would, in point of fact, draw contaminated groundwater from under Beatrice, along a highly permeable stratum of sand and coarse rock that lay under the riverbed. Nonetheless, Pinder told Schlichtmann, he wouldn't know for certain until he saw the field data from the EPA pump test. Then, on December 4, when the EPA activated the city wells and started the pump test, Pinder stationed himself at a monitoring well on the Beatrice property. He saw the water level at this monitoring well decline more than a foot in four hours, exactly the amount his computer model had predicted.

As far as Pinder was concetned, this and similar measurements from other monitoring wells proved beyond a doubt that groundwater from Beatrice was drawn under the river and into the city wells. It also proved that the Aberjona River played almost no role in satisfying the demand of the wells for water. Pinder reasoned that the thick layer of peat that formed the riverbed—twenty or more feet of decomposed leaves, roots, and branches—acted as a nearly impermeable lining. The river, in other words, was not a barrier. It wasn't even relevant.

This all seemed reasonable to Schlichtmann. But one detail troubled him. He and Pinder had both seen the ice on the river's surface that December. After the wells began pumping, the river's surface grew steadily lower, leaving shards of ice along the bank. Obviously the river had lost water. If it wasn't going to the wells, then where was it going?

At the Ritz-Carlton, Schlichtmann tried to get an answer to this question. Pinder had several explanations. Some water had been lost to evaporation. And some of it was being slowly drawn out of the river by the pumping action of the wells. But Pinder felt certain, based on the thickness of the peat layer, that it would take ten to twenty years for any river water to reach the wells.

It still didn't make sense to Schlichtmann. The river, he pointed out, had declined by six inches. That seemed like a lot of water. Pinder's explanations would not account for that much water.

Pinder, himself troubled now, agteed that this was true.

So where had the water gone? Schlichtmann asked.

Pinder didn't know.

They worked until after midnight, but Pinder could not come up with an explanation for the missing river water. He was tired and it was late. He insisted on going to bed. He wanted to have his wits about him tomorrow. He didn't want to face Facher without getting a good night's sleep.

Schlichtmann wouldn't leave. "We've got to figure this out, George. Let's go over it one more time."

"No," said Pinder stubbornly. "I'm going to sleep right now."

Schlichtmann, just as stubborn, refused to go.

"If you don't leave me alone," said Pinder angrily, "I'm going back to Princeton tomorrow morning."

Schlichtmann departed, feeling very worried.

Schlichtmann was waiting apprehensively when Pinder walked into the office early the next morning. He saw at once that their spat of last night had been forgotten. Pinder looked confident and happy. "In a moment of brilliance this morning, Jan, I figured out the river," Pinder said. "I don't know why it didn't occur to me before. It's really very obvious."

Tables

Table 1. Values of the well function W(u)

u	10^{-15}	10^{-14}	10^{-13}	10^{-12}	10^{-11}	10^{-10}	10^{-9}	10^{-8}
1	33.9616	31.6590	29.3564	27.0538	24.7512	22.4486	20.1461	17.8435
2	33.2684	30.9658	28.6632	26.3607	24.0581	21.7555	19.4529	17.1503
3	32.8629	30.5604	28.2578	25.9552	23.6526	21.3500	19.0474	16.7449
4	32.5753	30.2727	27.9701	25.6675	23.3649	21.0623	18.7598	16.4572
5	32.3521	30.0495	27.7470	25.4444	23.1418	20.8392	18.5366	16.2340
6	32.1698	29.8672	27.5646	25.2620	22.9595	20.6569	18.3543	16.0517
7	32.0157	29.7131	27.4105	25.1079	22.8053	20.5027	18.2001	15.8976
8	31.8821	29.5795	27.2769	24.9744	22.6718	20.3692	18.0666	15.7640
9	31.7643	29.4618	27.1592	24.8566	22.5540	20.2514	17.9488	15.6462

u	10^{-7}	10^{-6}	10^{-5}	10^{-4}	10^{-3}	10^{-2}	10^{-1}	10^{0}
1	15.5409	13.2383	10.9357	8.6332	6.3315	4.0379	1.8229	0.2194
2	14.8477	12.5451	10.2426	7.9402	5.6394	3.3547	1.2227	0.0489
3	14.4423	12.1397	9.8371	7.5348	5.2349	2.9591	0.9057	0.0130
4	14.1546	11.8520	9.5495	7.2472	4.9482	2.6813	0.7024	0.0038
5	13.9314	11.6289	9.3263	7.0242	4.7261	2.4679	0.5598	0.0011
6	13.7491	11.4465	9.1440	6.8420	4.5448	2.2953	0.4544	3.60e-4
7	13.5950	11.2924	8.9899	6.6879	4.3916	2.1508	0.3738	1.16e-4
8	13.4614	11.1589	8.8563	6.5545	4.2591	2.0269	0.3106	0.38e-4
9	13.3437	11.0411	8.7386	6.4368	4.1423	1.9187	0.2602	0.12e-4

Table 2. SCS curve numbers for selected agricultural and suburban land use

Land Use Description	Infiltration High	Good	Fair	Poor
Cultivated land	72	81	88	91
Pasture or range land: poor condition	68	79	86	89
Pasture or range land: good condition	39	61	74	80
Meadow	30	58	71	78
Wood or forest land: poor cover	45	66	77	83
Wood or forest land: good cover	25	55	70	77
Open spaces: good condition	39	61	74	80
Open spaces: fair condition	49	69	79	84

Table 3. Cumulative probability of the standard normal distribution $F(z)$.

z	0.000	0.005	0.010	0.015	0.020	0.025	0.030	0.035	0.040	0.045	0.050	0.055	0.060	0.065	0.070	0.075	0.080	0.085	0.090	0.095
0.0	0.5000	0.5020	0.5040	0.5060	0.5080	0.5100	0.5120	0.5140	0.5160	0.5179	0.5199	0.5219	0.5239	0.5259	0.5279	0.5299	0.5319	0.5339	0.5359	0.5378
0.1	0.5398	0.5418	0.5438	0.5458	0.5478	0.5497	0.5517	0.5537	0.5557	0.5576	0.5596	0.5616	0.5636	0.5655	0.5675	0.5695	0.5714	0.5734	0.5753	0.5773
0.2	0.5793	0.5812	0.5832	0.5851	0.5871	0.5890	0.5910	0.5929	0.5948	0.5968	0.5987	0.6006	0.6026	0.6045	0.6064	0.6083	0.6103	0.6122	0.6141	0.6160
0.3	0.6179	0.6198	0.6217	0.6236	0.6255	0.6274	0.6293	0.6312	0.6331	0.6350	0.6368	0.6387	0.6406	0.6424	0.6443	0.6462	0.6480	0.6499	0.6517	0.6536
0.4	0.6554	0.6573	0.6591	0.6609	0.6628	0.6646	0.6664	0.6682	0.6700	0.6718	0.6736	0.6754	0.6772	0.6790	0.6808	0.6826	0.6844	0.6862	0.6879	0.6897
0.5	0.6915	0.6932	0.6950	0.6967	0.6985	0.7002	0.7019	0.7037	0.7054	0.7071	0.7088	0.7106	0.7123	0.7140	0.7157	0.7174	0.7190	0.7207	0.7224	0.7241
0.6	0.7257	0.7274	0.7291	0.7307	0.7324	0.7340	0.7357	0.7373	0.7389	0.7405	0.7422	0.7438	0.7454	0.7470	0.7486	0.7502	0.7517	0.7533	0.7549	0.7565
0.7	0.7580	0.7596	0.7611	0.7627	0.7642	0.7658	0.7673	0.7688	0.7704	0.7719	0.7734	0.7749	0.7764	0.7779	0.7794	0.7808	0.7823	0.7838	0.7852	0.7867
0.8	0.7881	0.7896	0.7910	0.7925	0.7939	0.7953	0.7967	0.7981	0.7995	0.8009	0.8023	0.8037	0.8051	0.8065	0.8078	0.8092	0.8106	0.8119	0.8133	0.8146
0.9	0.8159	0.8173	0.8186	0.8199	0.8212	0.8225	0.8238	0.8251	0.8264	0.8277	0.8289	0.8302	0.8315	0.8327	0.8340	0.8352	0.8365	0.8377	0.8389	0.8401
1.0	0.8413	0.8426	0.8438	0.8449	0.8461	0.8473	0.8485	0.8497	0.8508	0.8520	0.8531	0.8543	0.8554	0.8566	0.8577	0.8588	0.8599	0.8610	0.8621	0.8632
1.1	0.8643	0.8654	0.8665	0.8676	0.8686	0.8697	0.8708	0.8718	0.8729	0.8739	0.8749	0.8760	0.8770	0.8780	0.8790	0.8800	0.8810	0.8820	0.8830	0.8840
1.2	0.8849	0.8859	0.8869	0.8878	0.8888	0.8897	0.8907	0.8916	0.8925	0.8934	0.8944	0.8953	0.8962	0.8971	0.8980	0.8988	0.8997	0.9006	0.9015	0.9023
1.3	0.9032	0.9041	0.9049	0.9057	0.9066	0.9074	0.9082	0.9091	0.9099	0.9107	0.9115	0.9123	0.9131	0.9139	0.9147	0.9154	0.9162	0.9170	0.9177	0.9185
1.4	0.9192	0.9200	0.9207	0.9215	0.9222	0.9229	0.9236	0.9244	0.9251	0.9258	0.9265	0.9272	0.9279	0.9285	0.9292	0.9299	0.9306	0.9312	0.9319	0.9325
1.5	0.9332	0.9338	0.9345	0.9351	0.9357	0.9364	0.9370	0.9376	0.9382	0.9388	0.9394	0.9400	0.9406	0.9412	0.9418	0.9424	0.9429	0.9435	0.9441	0.9446
1.6	0.9452	0.9458	0.9463	0.9468	0.9474	0.9479	0.9484	0.9490	0.9495	0.9500	0.9505	0.9510	0.9515	0.9520	0.9525	0.9530	0.9535	0.9540	0.9545	0.9550
1.7	0.9554	0.9559	0.9564	0.9568	0.9573	0.9577	0.9582	0.9586	0.9591	0.9595	0.9599	0.9604	0.9608	0.9612	0.9616	0.9621	0.9625	0.9629	0.9633	0.9637
1.8	0.9641	0.9645	0.9649	0.9652	0.9656	0.9660	0.9664	0.9667	0.9671	0.9675	0.9678	0.9682	0.9686	0.9689	0.9693	0.9696	0.9699	0.9703	0.9706	0.9710
1.9	0.9713	0.9716	0.9719	0.9723	0.9726	0.9729	0.9732	0.9735	0.9738	0.9741	0.9744	0.9747	0.9750	0.9753	0.9756	0.9759	0.9761	0.9764	0.9767	0.9770
2.0	0.9772	0.9775	0.9778	0.9780	0.9783	0.9786	0.9788	0.9791	0.9793	0.9796	0.9798	0.9801	0.9803	0.9805	0.9808	0.9810	0.9812	0.9815	0.9817	0.9819
2.1	0.9821	0.9824	0.9826	0.9828	0.9830	0.9832	0.9834	0.9836	0.9838	0.9840	0.9842	0.9844	0.9846	0.9848	0.9850	0.9852	0.9854	0.9856	0.9857	0.9859
2.2	0.9861	0.9863	0.9864	0.9866	0.9868	0.9870	0.9871	0.9873	0.9875	0.9876	0.9878	0.9879	0.9881	0.9882	0.9884	0.9885	0.9887	0.9888	0.9890	0.9891
2.3	0.9893	0.9894	0.9896	0.9897	0.9898	0.9900	0.9901	0.9902	0.9904	0.9905	0.9906	0.9907	0.9909	0.9910	0.9911	0.9912	0.9913	0.9915	0.9916	0.9917
2.4	0.9918	0.9919	0.9920	0.9921	0.9922	0.9923	0.9925	0.9926	0.9927	0.9928	0.9929	0.9930	0.9931	0.9931	0.9932	0.9933	0.9934	0.9935	0.9936	0.9937
2.5	0.9938	0.9939	0.9940	0.9940	0.9941	0.9942	0.9943	0.9944	0.9945	0.9945	0.9946	0.9947	0.9948	0.9948	0.9949	0.9950	0.9951	0.9951	0.9952	0.9953
2.6	0.9953	0.9954	0.9955	0.9955	0.9956	0.9957	0.9957	0.9958	0.9959	0.9959	0.9960	0.9960	0.9961	0.9962	0.9962	0.9963	0.9963	0.9964	0.9964	0.9965
2.7	0.9965	0.9966	0.9966	0.9967	0.9967	0.9968	0.9968	0.9969	0.9969	0.9970	0.9970	0.9971	0.9971	0.9972	0.9972	0.9972	0.9973	0.9973	0.9974	0.9974
2.8	0.9974	0.9975	0.9975	0.9976	0.9976	0.9976	0.9977	0.9977	0.9977	0.9978	0.9978	0.9978	0.9979	0.9979	0.9979	0.9980	0.9980	0.9980	0.9981	0.9981
2.9	0.9981	0.9982	0.9982	0.9982	0.9982	0.9983	0.9983	0.9983	0.9984	0.9984	0.9984	0.9984	0.9985	0.9985	0.9985	0.9985	0.9986	0.9986	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9987	0.9987	0.9988	0.9988	0.9988	0.9988	0.9988	0.9989	0.9989	0.9989	0.9989	0.9989	0.9989	0.9990	0.9990	0.9990	0.9990

N.B.: $F(-z) = 1 - F(|z|)$

Table 5. Frequency Factor for Pearson Type III distribution (negative skew)

$C_s \backslash T$	2	5	10	20	25	30	40	50	55	60	65	70	75	80	85	90	95	100	150	200
0.0	0.000	0.842	1.282	1.645	1.751	1.834	1.960	2.054	2.093	2.128	2.160	2.189	2.216	2.241	2.265	2.287	2.307	2.326	2.475	2.576
-0.1	0.017	0.846	1.270	1.616	1.716	1.794	1.912	2.000	2.036	2.069	2.099	2.126	2.151	2.174	2.196	2.216	2.235	2.253	2.389	2.482
-0.2	0.033	0.850	1.258	1.586	1.680	1.753	1.864	1.945	1.979	2.009	2.037	2.062	2.085	2.106	2.126	2.145	2.162	2.178	2.304	2.388
-0.3	0.050	0.853	1.245	1.555	1.643	1.712	1.814	1.890	1.921	1.949	1.974	1.997	2.018	2.038	2.056	2.073	2.089	2.104	2.218	2.294
-0.4	0.067	0.855	1.231	1.524	1.606	1.669	1.764	1.834	1.862	1.888	1.911	1.932	1.951	1.969	1.986	2.001	2.016	2.029	2.132	2.201
-0.5	0.083	0.857	1.216	1.491	1.567	1.626	1.714	1.777	1.803	1.827	1.848	1.867	1.884	1.901	1.915	1.929	1.942	1.955	2.047	2.108
-0.6	0.099	0.857	1.200	1.458	1.528	1.583	1.663	1.720	1.744	1.765	1.784	1.801	1.817	1.832	1.845	1.858	1.869	1.880	1.962	2.016
-0.7	0.116	0.857	1.183	1.423	1.489	1.538	1.611	1.663	1.685	1.704	1.721	1.736	1.750	1.763	1.775	1.786	1.797	1.806	1.879	1.926
-0.8	0.132	0.856	1.166	1.389	1.448	1.493	1.559	1.606	1.625	1.642	1.657	1.671	1.683	1.695	1.705	1.715	1.724	1.733	1.796	1.837
-0.9	0.148	0.854	1.147	1.353	1.407	1.448	1.507	1.549	1.566	1.581	1.594	1.606	1.617	1.627	1.636	1.645	1.653	1.660	1.714	1.749
-1.0	0.164	0.852	1.128	1.317	1.366	1.403	1.455	1.492	1.507	1.520	1.531	1.542	1.551	1.560	1.568	1.575	1.582	1.588	1.635	1.664
-1.1	0.180	0.848	1.107	1.280	1.324	1.357	1.403	1.435	1.448	1.459	1.469	1.478	1.487	1.494	1.501	1.507	1.513	1.518	1.557	1.581
-1.2	0.195	0.844	1.086	1.243	1.282	1.311	1.352	1.379	1.390	1.400	1.408	1.416	1.423	1.429	1.435	1.440	1.445	1.449	1.482	1.501
-1.3	0.210	0.838	1.064	1.206	1.240	1.265	1.300	1.324	1.333	1.342	1.349	1.355	1.361	1.366	1.371	1.375	1.379	1.383	1.409	1.424
-1.4	0.225	0.832	1.041	1.168	1.198	1.220	1.250	1.270	1.278	1.285	1.290	1.296	1.300	1.305	1.308	1.312	1.315	1.318	1.339	1.351
-1.5	0.240	0.825	1.018	1.131	1.157	1.175	1.201	1.217	1.224	1.229	1.234	1.238	1.242	1.245	1.248	1.251	1.254	1.256	1.272	1.282
-1.6	0.254	0.817	0.994	1.093	1.116	1.131	1.152	1.166	1.171	1.175	1.179	1.183	1.186	1.188	1.191	1.193	1.195	1.197	1.209	1.216
-1.7	0.268	0.808	0.970	1.056	1.075	1.088	1.105	1.116	1.120	1.124	1.127	1.130	1.132	1.134	1.136	1.138	1.139	1.140	1.150	1.155
-1.8	0.281	0.799	0.945	1.020	1.035	1.046	1.060	1.069	1.072	1.075	1.077	1.079	1.081	1.082	1.084	1.085	1.086	1.087	1.094	1.097
-1.9	0.294	0.788	0.920	0.984	0.997	1.005	1.016	1.023	1.026	1.028	1.029	1.031	1.032	1.033	1.034	1.035	1.036	1.037	1.042	1.044
-2.0	0.307	0.777	0.895	0.949	0.959	0.966	0.975	0.980	0.982	0.983	0.984	0.986	0.987	0.987	0.988	0.989	0.989	0.990	0.993	0.995
-2.1	0.319	0.765	0.869	0.915	0.923	0.928	0.935	0.939	0.940	0.941	0.942	0.943	0.944	0.944	0.945	0.945	0.946	0.946	0.948	0.949
-2.2	0.330	0.752	0.844	0.882	0.888	0.892	0.897	0.900	0.901	0.902	0.903	0.903	0.904	0.904	0.904	0.905	0.905	0.905	0.907	0.907
-2.3	0.341	0.739	0.819	0.850	0.855	0.858	0.862	0.864	0.864	0.865	0.865	0.866	0.866	0.866	0.867	0.867	0.867	0.867	0.868	0.869
-2.4	0.351	0.725	0.795	0.819	0.823	0.826	0.828	0.830	0.830	0.830	0.831	0.831	0.831	0.831	0.832	0.832	0.832	0.832	0.833	0.833
-2.5	0.360	0.711	0.771	0.790	0.793	0.795	0.797	0.798	0.798	0.798	0.798	0.799	0.799	0.799	0.799	0.799	0.799	0.799	0.800	0.800
-2.6	0.369	0.696	0.747	0.762	0.765	0.766	0.767	0.768	0.768	0.768	0.768	0.768	0.769	0.769	0.769	0.769	0.769	0.769	0.769	0.769
-2.7	0.376	0.681	0.724	0.736	0.738	0.739	0.739	0.740	0.740	0.740	0.740	0.740	0.740	0.740	0.740	0.740	0.740	0.740	0.741	0.741
-2.8	0.384	0.666	0.702	0.711	0.712	0.713	0.713	0.714	0.714	0.714	0.714	0.714	0.714	0.714	0.714	0.714	0.714	0.714	0.714	0.714
-2.9	0.390	0.651	0.681	0.688	0.688	0.689	0.689	0.689	0.689	0.689	0.689	0.690	0.690	0.690	0.690	0.690	0.690	0.690	0.690	0.690
-3.0	0.396	0.636	0.660	0.665	0.666	0.666	0.666	0.666	0.667	0.667	0.667	0.667	0.667	0.667	0.667	0.667	0.667	0.667	0.667	0.667

Formulae

Formulae I

$$E = mc^2 \qquad F = ma \qquad PV = nRT$$

$$Q_x = -K_x A \frac{dh}{dx} \qquad q = \frac{Q}{A} \qquad v_s = \frac{q}{n}$$

$$K_e = \frac{d_1 + d_2}{\frac{d_1}{K_1} + \frac{d_2}{K_2}} \qquad K_e = \frac{K_1 d_1 + K_2 d_2}{d_1 + d_2}$$

$$Q_w = 2\pi BK \frac{h - h_1}{\ln(r/r_1)} \qquad Q_w = \pi K \frac{h^2 - h_1^2}{\ln(r/r_1)} \qquad s = \frac{Q_w}{2\pi T} \ln \frac{R}{r}$$

$$h^2 = h_0^2 + \left(h_L^2 - h_0^2\right)\frac{x}{L} + \frac{Nx}{k}(L - x)$$

$$s = \frac{Q_w}{4\pi T} \mathrm{W}(u) \qquad u = \frac{r^2 S}{4tT} \qquad s(r,t) = \frac{Q_w}{4\pi T} \ln\left(\frac{2.25tT}{Sr^2}\right)$$

$$F(t + \Delta t) = F(t) + k_o \Delta t + \psi_s \Delta\theta \ln\left(\frac{F(t + \Delta t) + \psi_s \Delta\theta}{F(t) + \psi_s \Delta\theta}\right)$$

$$F\left(t_p\right) = \frac{k_o \, \Delta\theta \, \psi_s}{i(t) - k_o} \qquad f = k_o\left(\frac{\Delta\theta \, \psi_s}{F} + 1\right) \qquad \Delta t_p = \frac{F(t_p) - F(t)}{i(t)}$$

$$F(t + \Delta t) = F\left(t_p\right) + k\left(\Delta t - \Delta t_p\right) + \psi_s \Delta\theta \ln\left(\frac{F(t + \Delta t) + \psi_s \Delta\theta}{F\left(t_p\right) + \psi_s \Delta\theta}\right)$$

$$S = k[xI + (1 - x)Q] \qquad Q_{i+1} = C_1 I_{i+1} + C_2 I_i + C_3 Q_i$$

$$C_1 = \frac{\Delta t - 2Kx}{2K(1 - x) + \Delta t} \qquad C_2 = \frac{\Delta t + 2Kx}{2K(1 - x) + \Delta t} \qquad C_3 = \frac{2K(1 - x) - \Delta t}{2K(1 - x) + \Delta t}$$

$$\frac{dS}{dt} = I(t) - Q(t) \qquad Q = C_w L h^{3/2} \qquad Q = C_d A_p \sqrt{2gh}$$

$$y_{n+1} = y_n + \frac{\Delta t}{2}(k_1 + k_2) \qquad k_1 = f(t_n, y_n) \qquad k_2 = f(t_n + \Delta t, y_n + k_1 \Delta t)$$

$$q_l = \exp(-at)\begin{cases} i_0\left[\exp(at) - \exp(at_0)\right] & t_0 \le t < t_1 \\ i_0\left[\exp(at_1) - \exp(at_0)\right] + i_1\left[\exp(at) - \exp(at_1)\right] & t_1 \le t < t_2 \end{cases}$$

Formulae II

$$P(X \geq x_T) = \frac{1}{T} \qquad T = \frac{n+1}{m} \qquad P(X \leq x) = F(x) = \int_{-\infty}^{x} f(u)du$$

$$\overline{x} = \frac{1}{n}\sum_{i=1}^{n} x_i \qquad s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \overline{x})^2 \qquad C_s = \frac{n}{(n-1)(n-2)s^3}\sum_{i=1}^{n}(x_i - \overline{x})^3$$

$$C_s = \frac{n^2\sum_{i=1}^{n}x^3 - 3n\sum_{i=1}^{n}x\sum_{i=1}^{n}x^2 + 2\left(\sum_{i=1}^{n}x\right)^3}{n(n-1)(n-2)s^3} \qquad \overline{C}_s = \left(1+\frac{6}{n}\right)C_s \qquad f_s(x_i) = \frac{n_i}{n}$$

$$r_d = \sum_{i=1}^{M} P_e = \sum_{i=1}^{M}(P - \phi\Delta t) \qquad Q = \frac{(P-0.2S)^2}{P+0.8S} \qquad f(x) = \frac{dF(x)}{dx}$$

$$x_T = \overline{x} + Ks \qquad T = \frac{n+1}{m} \qquad F = 1 - \frac{1}{T} \qquad T = \frac{n+1-2a}{m-a}$$

$$t_p = C_t(LL_c)^{0.3} \qquad T_b = 3 + t_p/8 \qquad T_b = 4t'_p$$

$$\binom{n}{x} = \frac{n!}{x!(n-x)!} \qquad F(x) = \sum_{i=0}^{x}\binom{n}{i}p^i(1-p)^{n-i} \qquad \overline{x} = np \qquad \sigma^2 = np(1-p)$$

$$f(x) = \lambda\exp(-\lambda x) \qquad F(X) = 1 - \exp(-\lambda X) \qquad \lambda = \frac{1}{\overline{x}}$$

$$T_B = T_R + 1.67T_R \qquad t_p = l^{0.8}(S+1)^{0.7}/1900\sqrt{y} \qquad T_R = D/2 + t_p$$

$$F(x) = \exp\left[-\exp\left(-\frac{x-u}{\alpha}\right)\right] \qquad \alpha = \frac{\sqrt{6}s}{\pi} \qquad u = \overline{x} - \gamma\alpha \qquad \gamma = 0.5772$$

$$t'_p = t_p + 0.25(D - D') \qquad Q_p = 640C_pA/t'_p \qquad Q_p = 484A/T_R$$

$$P_e = P - I_a - F_a \qquad F_a = \frac{S(P-I_a)}{P-I_a+S} \qquad I_a = 0.2S$$

$$Q_n = \sum_{m=1}^{n\leq M} P_m U_{n-m+1} \qquad D = t_p/5.5 \qquad F_s(x_i) = \sum_{j=1}^{i} f_s(x_j)$$

$$S = \frac{1000}{C_n} - 10 \qquad C_d = \frac{4.2C_n}{10 - 0.058C_n} \qquad C_w = \frac{23C_n}{10 + 0.13C_n}$$

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma}\exp\left[-\frac{(x-\mu)^2}{2\sigma^2}\right] \qquad z = \frac{x-\mu}{\sigma} \qquad F(z) = \frac{1}{2}\left(1 + \operatorname{erf}\left(\frac{z}{\sqrt{2}}\right)\right)$$